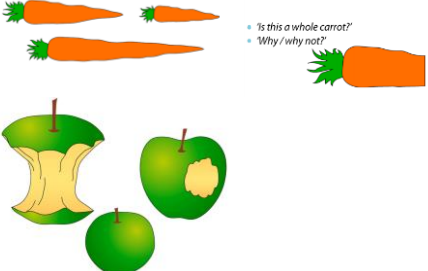
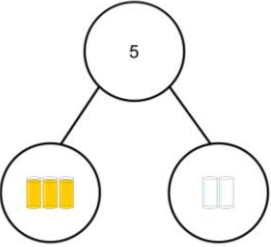
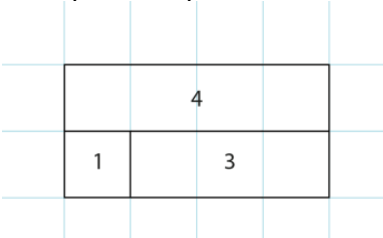
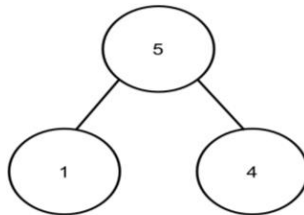
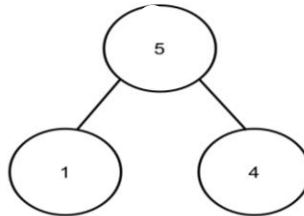
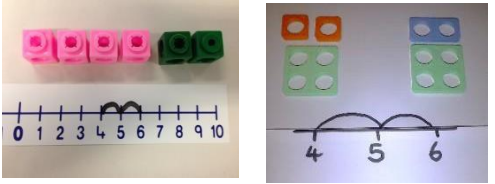
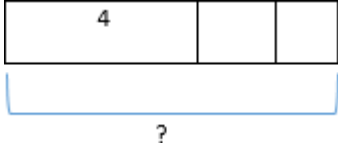
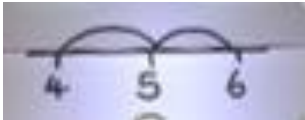


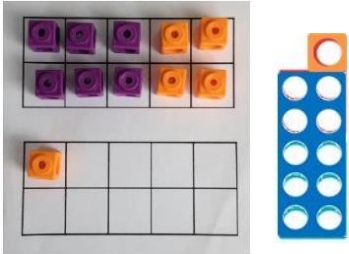
Maths Calculation Policy

Addition

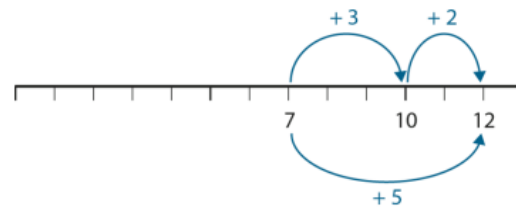
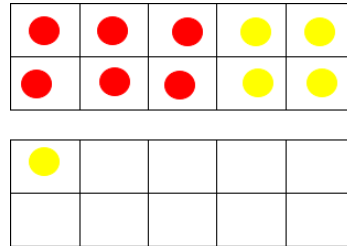
Key language which should be used: sum, total, parts and wholes, plus, add altogether, more than, is equal to, is the same as, addends

Concrete	Pictorial	Abstract
<p>Concept of part/whole. (what is a part/not a part? and what is a whole/not a whole?)</p>  <p>• Is this a whole carrot? • Why / why not?</p>	 <p>Using squared paper can help introduce the bar model to represent parts and whole.</p> 	 
<p>Counting on using number lines by using cubes or numicon</p> 	<p>A bar model which encourages the children to count on</p>  <p>Maths stories - First, there are four horses in field. Next, two more horses come into the field. How many horses are in the field now?</p>	<p>The abstract number line:</p> <p>What is 2 more than 4? What is the sum of 4 and 4? What's the total of 4 and 2?</p> <p>$4 + 2$</p> 

Regrouping to make 10 by using ten frames and counters/cubes or using numicon:
 $6 + 5$



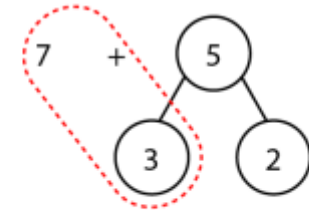
Children to draw the ten frame and counters/cubes



Children to develop an understanding of equality e.g. $6 + \square = 11$ and

$$6 + 5 = 5 + \square \quad 6 + 5 = \square + 4$$

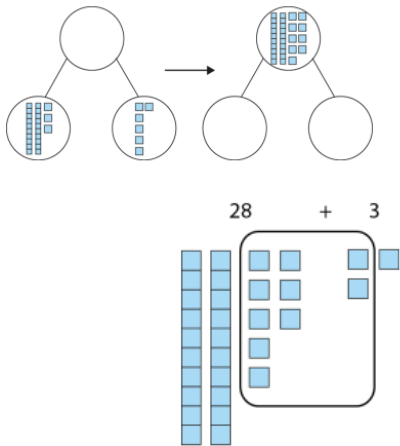
Mentally partitioning one of the addends to make a model of ten. Practice adding three one-digit numbers is key as a stepping stone to this strategy.



$$7 + 3 = 10$$

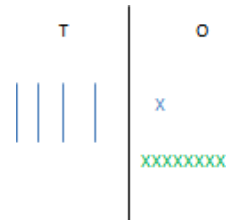
$$10 + 2 = 12$$

TO + O using base 10. Continue to develop understanding of partitioning, recombining and place value $41 + 8$

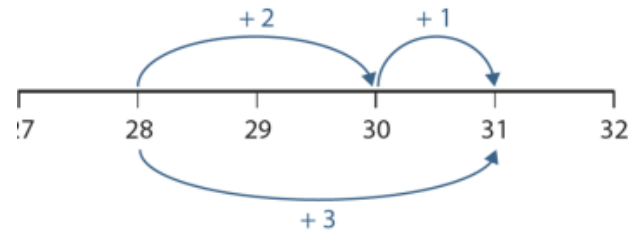


And bridging ten

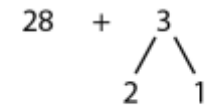
Children to represent the concrete using a symbol e.g. lines for tens and dot/crosses for ones. Variation in these symbols should be provided.



Use of a number line to represent when bridging ten.

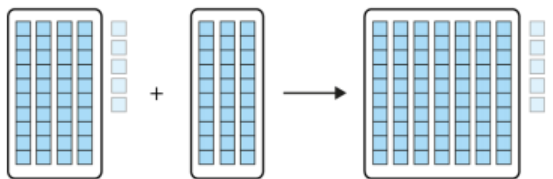


When bridging ten, partitioning the second addend.



$$\begin{aligned}
 28 + 3 &= 28 + 2 + 1 \\
 &= 30 + 1 \\
 &= 31
 \end{aligned}$$

TO + T using equipment such as base ten



Use of a 100 square Image of a 100 square.

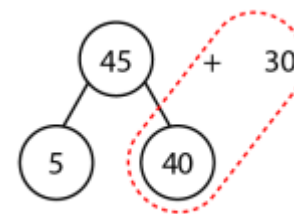
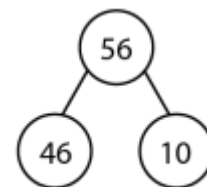
	1	2	3	4	5	6	7	8	9	10	
$14 + 10 = 24$	11	12	13	14	15	16	17	18	19	20	$24 - 10 = 14$
$24 + 10 = 34$	21	22	23	24	25	26	27	28	29	30	$34 - 10 = 24$
$34 + 10 = 44$	31	32	33	34	35	36	37	38	39	40	$44 - 10 = 34$
$44 + 10 = 54$	41	42	43	44	45	46	47	48	49	50	$54 - 10 = 44$
$54 + 10 = 64$	51	52	53	54	55	56	57	58	59	60	$64 - 10 = 54$
$64 + 10 = 74$	61	62	63	64	65	66	67	68	69	70	$74 - 10 = 64$
$74 + 10 = 84$	71	72	73	74	75	76	77	78	79	80	$84 - 10 = 74$
$84 + 10 = 94$	81	82	83	84	85	86	87	88	89	90	$94 - 10 = 84$
	91	92	93	94	95	96	97	98	99	100	$94 - 10 = 84$

Using known facts and procedural variation.

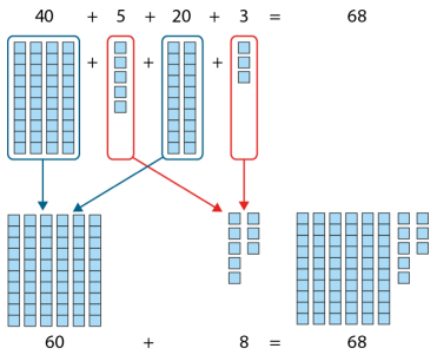
$$14 + 10 = 24$$

$$24 + 10 = 34$$

$$34 + 10 = 44$$



TO + TO using base 10. Continue to develop understanding of partitioning and place value and use this to support addition. Begin with no exchanging. $36 + 25$



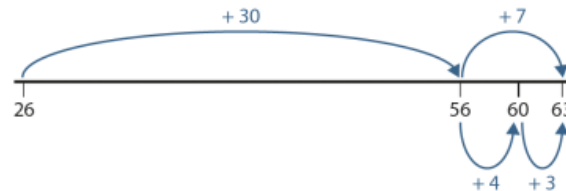
Adding two tens and two ones leading into adding two 2-digit numbers.

68			
40	5	20	3



Number line leads to just mental partitioning of second addend.

$26 + 30 + 7$

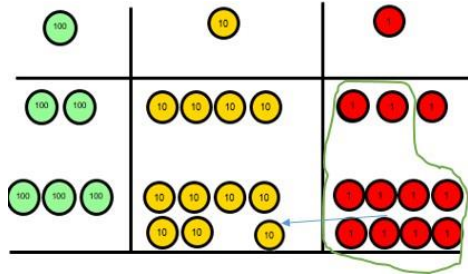


Partitioning both addends	Partitioning one addend
$\begin{array}{r} 26 \\ \swarrow \searrow \\ 20 \quad 6 \end{array} + \begin{array}{r} 37 \\ \swarrow \searrow \\ 30 \quad 7 \end{array}$	$26 + \begin{array}{r} 37 \\ \swarrow \searrow \\ 30 \quad 7 \end{array}$
$20 + 30 = 50$ $6 + 7 = 13$ $50 + 13 = 63$	$26 + 30 = 56$ $56 + 7 = 63$
so $\pounds 26 + \pounds 37 = \pounds 63$	

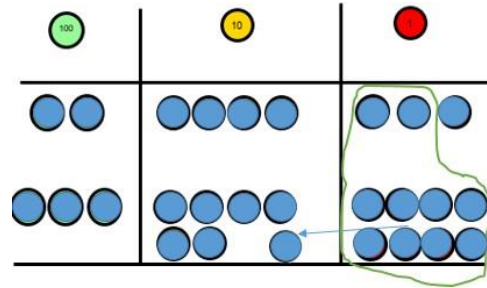
Choosing the most efficient method.

$$\begin{array}{r} 36 + \\ 45 \\ 1 \\ \hline \underline{81} \end{array}$$

Use of place value counters to add **HTO + TO**, **HTO + HTO** etc. once the children have had practice with this, they should be able to apply it to larger numbers and the abstract



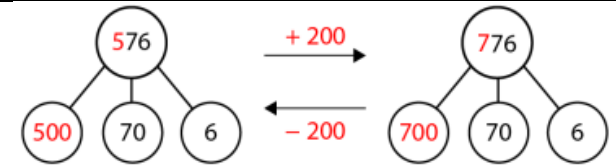
Children to represent the counters e.g. like the image below



If the children are completing a word problem, draw a bar model to represent what it's asking them to do

?	
243	368

When just adding hundreds, tens or ones, then use of a part whole model to aid fluency.



$$576 + 200 = 776$$

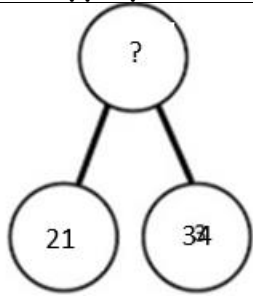
$$776 - 200 = 576$$

Link mental methods to subtraction

$$\begin{array}{r} 243 + \\ 368 \\ 11 \end{array}$$

$$\begin{array}{r} \hline 611 \end{array}$$

Fluency and variation. Asking different ways to solve addition calculations. Children are encouraged to make the most efficient choices where appropriate.



Sam saved £21 one week and £34 another. How much did he save in total?

$21 + 34 = 55$ Prove it. (reasoning but children need to be fluent in representing this).

Balancing equations
 $50 + 5 = 55 = 21 + \underline{\quad}$

Function machines

16	+ 30 =	
26		
36		

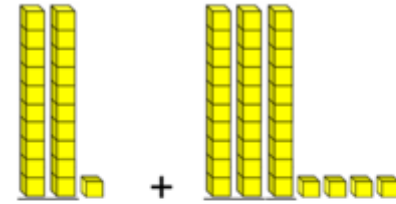
$21 + \underline{\quad} = 75 - 20$

2	1	+
3	4	
5	5	

Missing number
 $\underline{\quad} = 21 + 34$

$55 = 21 + \underline{\quad}$

What is the sum of twenty one and thirty four?






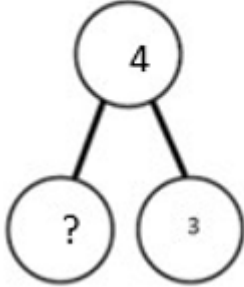


Always use missing digit problems too:

Tens		Ones
●	●	●
●	●	?
?		4

Subtraction-

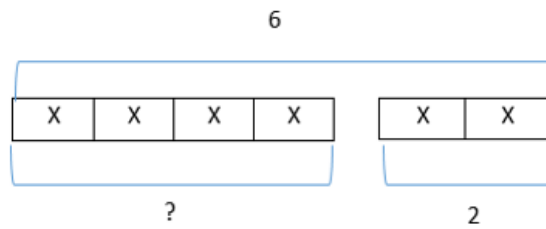
Key language which should be used: take away, less than, the difference, subtract, minus, fewer, decrease, 7 take away 3, the difference is four, How much more? How much less? How much fewer?

Concrete	Pictorial	Abstract
<p>Physically taking away and removing parts from a whole. This results in part being left.</p> <p>$4 - 3 = 1$</p>  <p>Playing number stories with equipment. E.g. Four lambs were in a field. Three lambs left the field. How many are left in the field?</p>	<p>Children to draw the concrete resources they are using and cross out.</p>  <p>Use of the bar model:</p>  <p>Recognising pictorial representations of a fingers model. $5 - 3 = 2$</p> 	<p>$4 - 3 =$ _____ = $4 - 3$</p>  

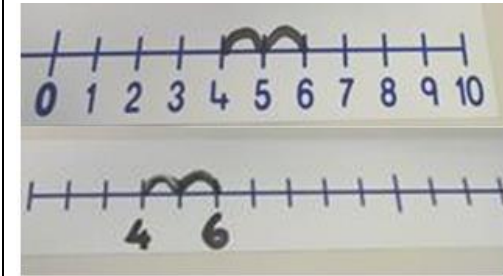
Counting back (using number lines or number tracks)



Children representing what they see pictorially. Links to bar model.



Using a number line to count backwards.



Finding the difference



2 cars



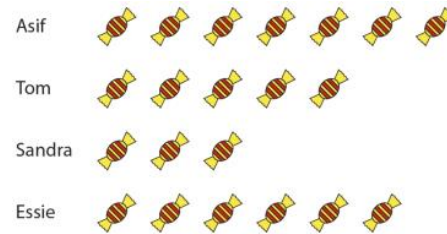
2 cars

Children draw representations of the equipment they have used.

Bar models showing the gap or difference

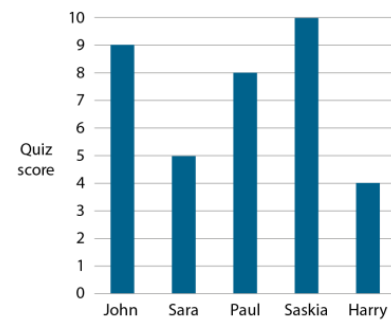


2 cars



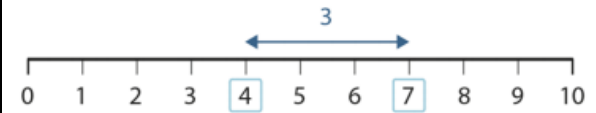
Interpreting pictograms and bar charts.

How many more sweets does Tom have than Sandra?



How many more points did Paul score than Sara?

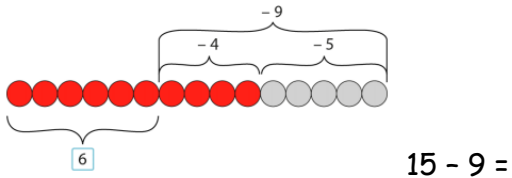
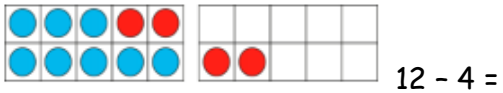
Using number lines to find or show the difference.



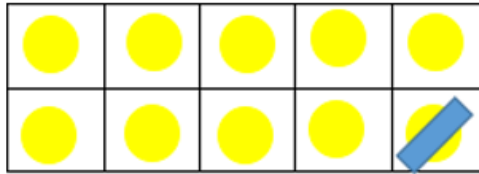
8 - 6, the difference is?

Children also explore why $9 - 7 = 8 - 6$.

Bridging ten

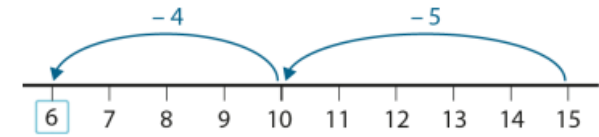


Children represent the tens frame pictorially with crossings out.



Pupils move to a more abstract number line and on to partitioning the subtrahend.

$$15 - 9 =$$

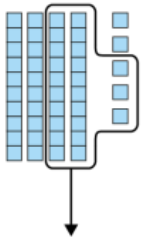


$$\begin{array}{r} 12 - 3 \\ \quad 2 \quad 1 \end{array}$$

The focus is on using knowledge of number bonds to subtract rather than counting back or on.

Subtracting using place value (without bridging)

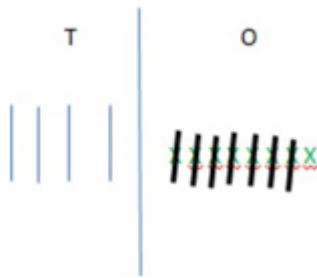
'taking away' 23



Subtracting 15



Drawing representations of tens and ones.



4	8	-
1	7	
3	1	

$$45 - 23 = 20 + 3$$

$$45 - 23 = 45 - 20 - 3$$

Missing number that shows understanding of the process.

$$84 - 12 = 84 - 2 - \square$$

$$68 - 23 = 68 - \square - 20$$

$$47 - 25 = 47 - \square - \square$$

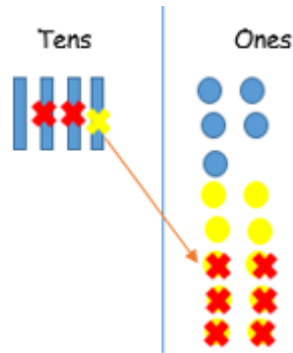
Subtracting using place value (bridging ten)



$45 - 26 =$

1. Start by partitioning 45.
2. Exchange one ten for ten ones.
3. Subtract the ones, then the tens.

Represent this pictorially. Moving on from this quickly to use an abstract method.

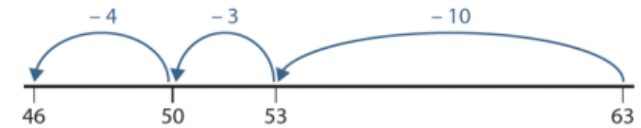


$$86 - 27 = \square$$

$\begin{array}{c} 27 \\ / \quad \backslash \\ 20 \quad 7 \\ \quad \quad / \quad \backslash \\ \quad \quad 6 \quad 1 \end{array}$

The second partitioning will probably be done mentally.

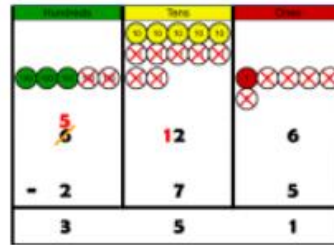
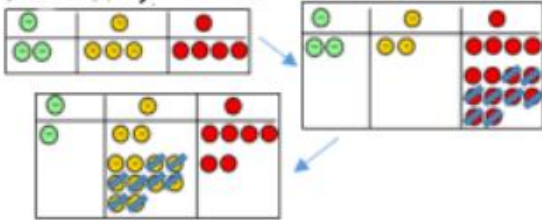
Number line. $63 - 17$



1 3	5	-
2	6	
1	9	

Continued methods for calculation including column method.

Column method (using place value counters) 234-88

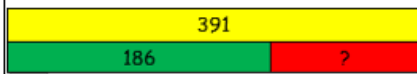
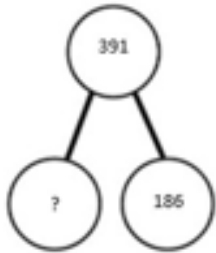


Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.

2 1	3 2	4	-
	8	8	
1	4	6	

Starting with just regrouping from tens to ones and develop this using small steps.

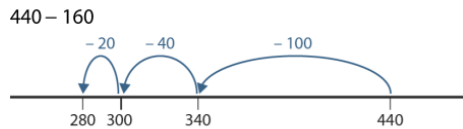
Fluency and variation. Different ways to ask children to solve subtraction problems. Always encourage children to make independent choices about the most efficient method for them to solve calculations



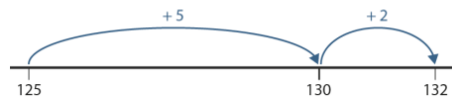
Function machines

3 2 1	-90 =	
4 6 7		
8 9 1		

Continued use of numberline.



- Finding the difference (working forward from the subtrahend) – more efficient



Raj spent £391. Timmy spent £186. How much more did Raj spend? How much fewer/less did Timmy spend?

I had 391 metres to run. After 186 I stopped. How many metres do I have left to run?

Multi step word problems that include subtraction in addition to other calculations.

$391 - 186 =$
 $\underline{\hspace{2cm}} = 391 - 186$

Find the difference between 391 and 186.

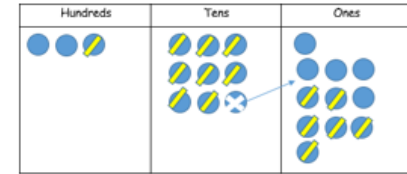
Subtract 186 from 391

What is 186 less than 391?

Balancing calculations

$150 - \underline{\hspace{1cm}} = 83 = 183 - \underline{\hspace{1cm}}$
 $46 - 20 = \underline{\hspace{1cm}} + 21$

What is the calculation in this place value chart?



What could the missing numbers be? Is there more than one solution?

$$\begin{array}{r} 39\Box \\ - \Box\Box6 \\ \hline \Box05 \end{array}$$

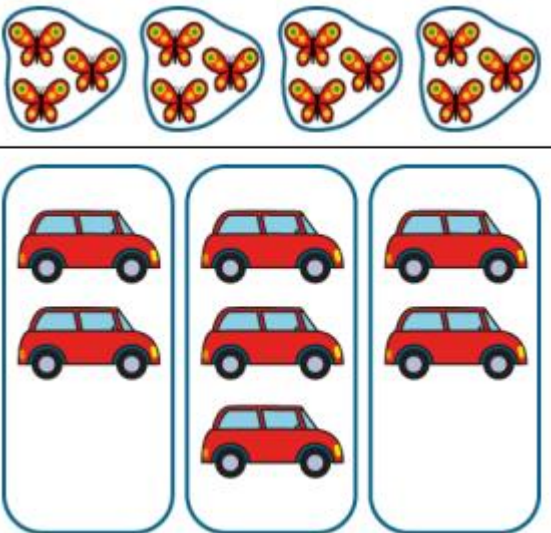
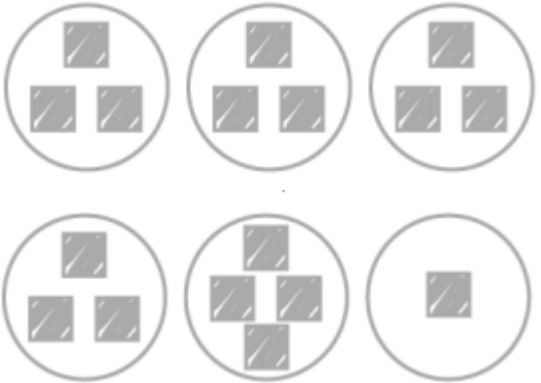
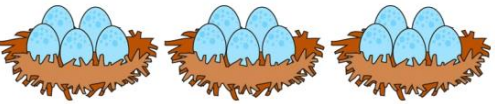
Subtracting more than one number.

65		
30	16	?

$65 - 30 - 16 = \square$

Multiplication-

Key language which should be used: double times, multiplied by, the product of, groups of, lots of, is equal to, is the same as, twice as big, arrays, factors

Concrete	Pictorial	Abstract
<p data-bbox="107 335 716 367">Understanding of equal and unequal groups</p>  <p>The concrete models consist of two rows. The top row shows four groups, each enclosed in a blue outline and containing five orange butterflies. The bottom row shows three groups, each enclosed in a blue outline and containing three red cars. The first and third groups in the bottom row have two cars each, while the middle group has three cars.</p>	<p data-bbox="790 335 1361 406">Drawing of equal and unequal groups from concrete models</p>  <p>The pictorial models consist of two rows of circles. The top row shows three circles, each containing three grey squares arranged in a triangle (one on top, two below). The bottom row shows three circles: the first contains three grey squares in a triangle, the second contains four grey squares in a cross shape (one in the center, three around it), and the third contains one grey square.</p>	<p data-bbox="1473 335 2116 406">Using stem sentences to describe the grouping concept.</p>  <p>The abstract models show three nests, each containing three blue eggs.</p> <ul data-bbox="1478 526 1825 614" style="list-style-type: none">• There are ___ equal groups of eggs.• There are ___ eggs in each group.• There are ___ groups of ___.

Repeated grouping/repeated addition
(With lots of different types of equipment)

3×4 three groups of four/lots of four
 4×3 a group of four, three times.

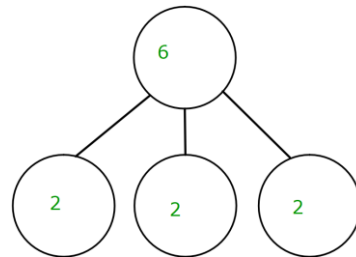
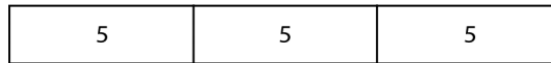
$4 + 4 + 4 =$



Children to represent the practical resources in a picture e.g.

xx xx xx
xx xx xx

Use of bar models and part whole models for a more structured method.



$3 \times 4 = \underline{\quad}$

$4 + 4 + 4 = \underline{\quad}$

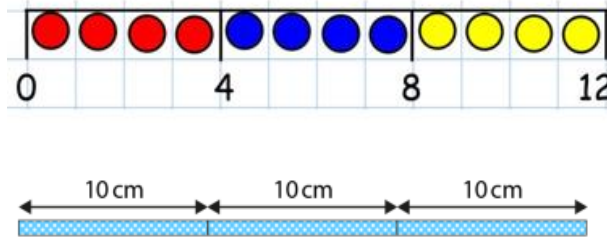
$\underline{\quad} = 4 \times 3$

There are four cubes in each group.
There are three groups.
There are 12 cubes altogether.
Three groups of four cubes are equal to 12 cubes

Use number lines to show repeated groups



Represent multiplication stories pictorially alongside a number line.

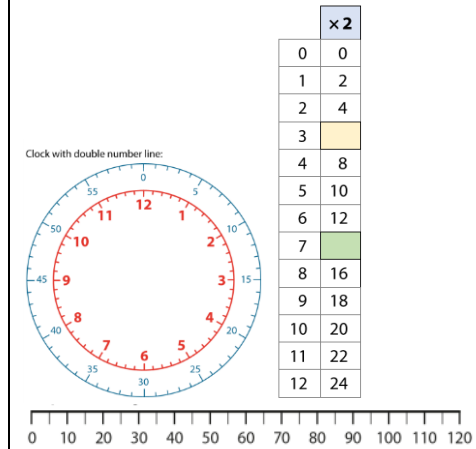
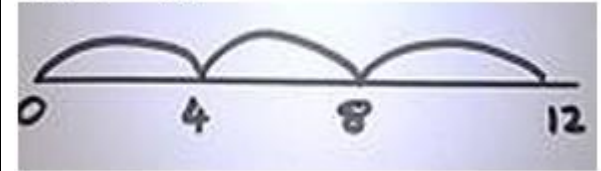


Using pictures of measurement, both to scale and not to scale.

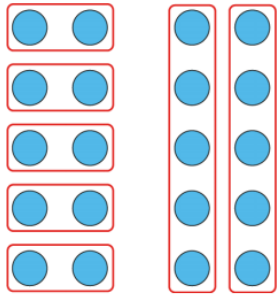
Include scaling problems e.g. the ribbon is three times as long. The table is twice as heavy.

Abstract number lines include vertical and scales.

$$3 \times 4 = 12$$



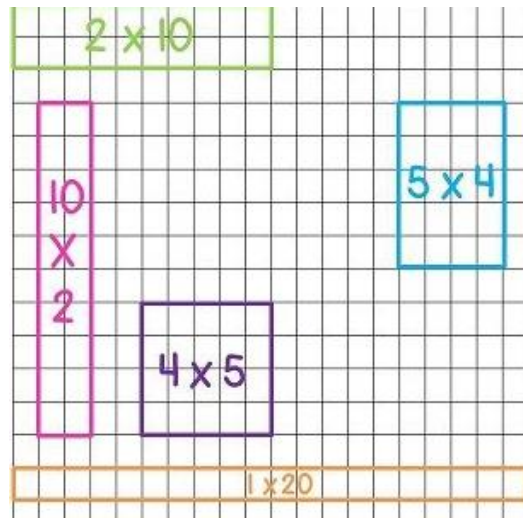
Use arrays to illustrate commutativity
(counters and many other objects should also be used.)



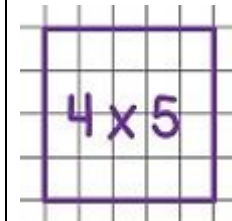
Finding and interpreting arrays in and around school.



Draw arrays to demonstrate commutativity and solve problems.



Children can use the arrays to write a range of calculations.



Eg. $4 \times 5 = 20$
 $5 \times 4 = 20$
 $4 + 4 + 4 + 4 + 4 = 20$
 $5 + 5 + 5 + 5 = 20$

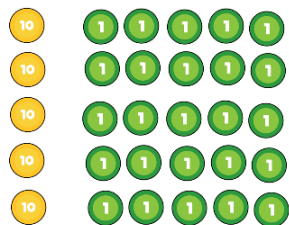
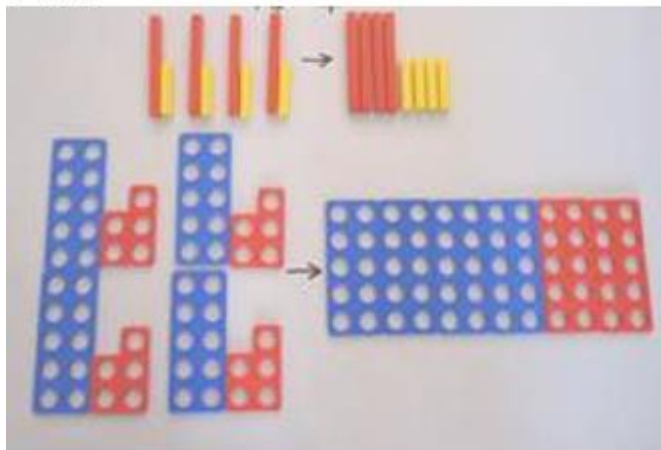
Include missing number.

$$_ + _ + _ + _ = 20$$

$$20 = _ \times 5$$

Partitioning to multiply
(use a variety of resources such as numicom, base 10, cuisinaire rods and place value counters).

4×15



$15 \times 4 =$

Tens Ones



$4 \times 10 = 40$

$4 \times 5 = 20$

$40 + 20 = 60$

This can also be drawn as a bar model.

$32 \times 3 =$

90	30	2	6
	30	2	
	30	2	

Children to be encouraged to show and explain the steps they have taken.

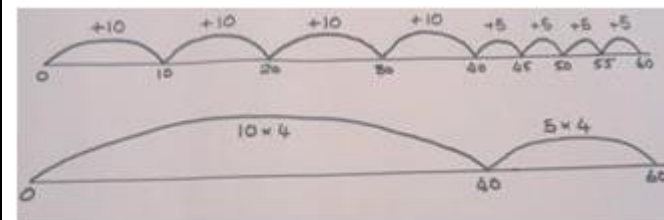
$15 \times 4 =$
 $10 \quad 5$

$4 \times 10 = 40$

$4 \times 5 = 20$

$40 + 20 = 60$

A numberline can also be used.



Encourage use of known number facts to calculate.

Formal methods of multiplication

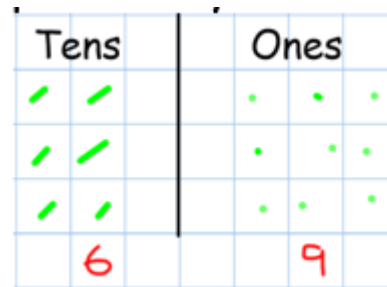
With place value counters or base 10

$$3 \times 23$$

Make 23, 3 times. See how many ones, then how many tens



Children to represent the counters in a pictorial way.



Children to record what it is they are doing to show understanding.

$$\begin{array}{l} 23 \times 3 = \\ \swarrow \searrow \\ 20 \quad 3 \end{array} \quad \begin{array}{l} 3 \times 3 = 9 \\ 3 \times 20 = 60 \\ 9 + 60 = 69 \end{array}$$

2	3	x		
	3			
<hr/>				
	9	(3 x 3)		
6	0	(20 x 3)		
<hr/>				
6	9			

The concrete model is briefly acknowledged, but children should be ready to rapidly grasp the written method.

$$6 \times 23$$

Step 1: get 6 lots of 23



Step 2: 6×3 is 18. Can I make an exchange? Yes! Ten ones for one ten....

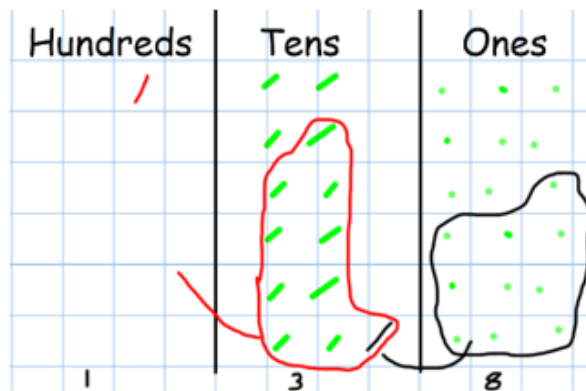


Step 3: 6×2 tens and my extra ten is 13 tens. Can I make an exchange? Yes! Ten tens for one hundred...

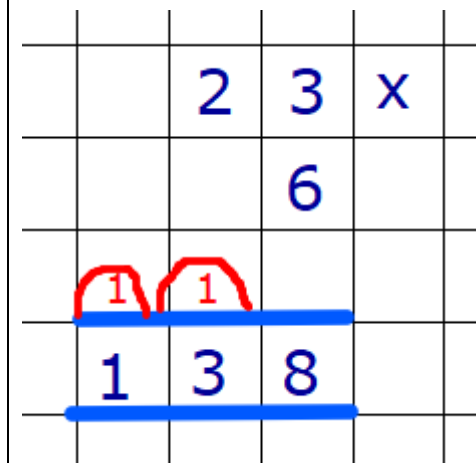


Step 4- what do I have I each column?

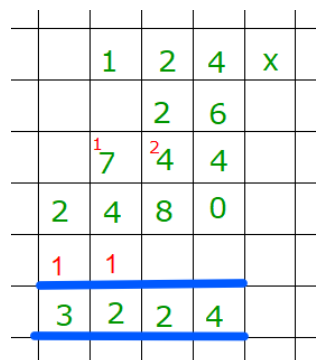
Again the pictorial method is briefly acknowledged, but children should be ready to rapidly grasp the written method by this stage.



The aim is to achieve the formal method, but ensuring that the children need to understand how it works.



When children start to multiply $3d \times 3d$ and $4d \times 2d$ etc, they should be confident with the abstract formal method.



Fluency variation, Different ways to ask children to solve calculations. Always encourage children to make independent choices about the most efficient method for them to solve calculations

23	23	23	23	23	23

?

Use counters, prove that $6 \times 23 = 138$.

Why is $6 \times 23 = 32 \times 6$?

Function machines

3	X 60	
4		
7		

Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week?

Tom saved 23p three days a week. How much did he save in 2 weeks, 3 weeks? Etc.

Sarah water bottle had 23ml of water left. Tim had six times as much water as Sarah. How many ml of water did Tim have?

Multi step word problems including a mix of different calculation types.

Missing number problems

$$80 \times \underline{\quad} = 7200$$

$$420 = 7 \times \underline{\quad}$$

$$23 \times 6 = 23 \times \underline{\quad} \times \underline{\quad}$$

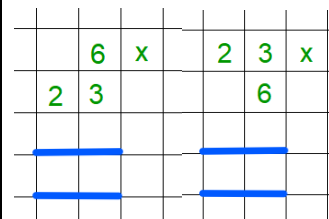
$$23 \times 6 = \underline{\quad} \times 6 + \underline{\quad} \times 6$$

Find the product of 6 and 23.

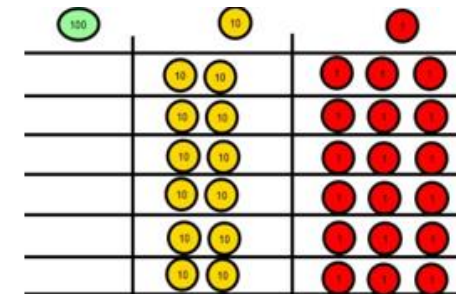
What number are 6 and 23 factors of?

$$\underline{\quad} = 6 \times 23$$

$$6 \times 23 = \underline{\quad}$$

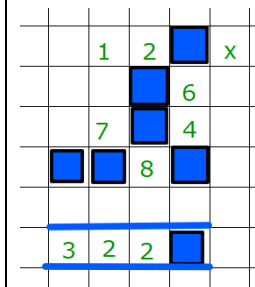


What is the calculation?
What is the answer?



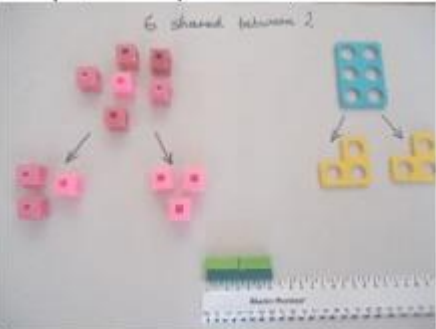
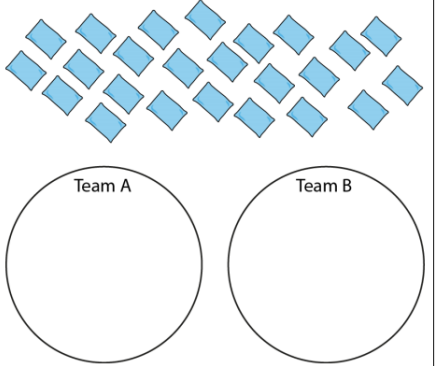

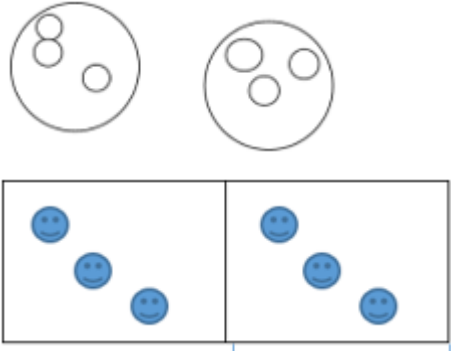
$\underline{\quad}, \underline{\quad}, 18, 24, \underline{\quad}, \underline{\quad}, 64$

$$8 \times \underline{\quad} = 32 \times \underline{\quad}$$



Division-

Key language which should be used: share, group, divide, divided by, half, is equal to, is the same as, split into equal groups of, shared into ___ groups.

Concrete	Pictorial	Abstract		
<p>Division as sharing (Many concrete objects can be used e.g. children and hoops, teddy bears, cakes and plates etc.)</p>  <p><i>There are twenty-four bean bags. If they are shared equally between two teams, how many bean bags does each team get?</i></p> 	<p>Children draw pictures to represent the concrete when solving a problem.</p> <p>E.g. Six sweets are shared equally between 3 children.</p> <p>Child: 1 2 3</p>  <p>This can also be done in circles or in a bar model so that all four operations have a similar structure and links can be made between the two.</p> 	<p>$6 \div 2 = 3$</p> <table border="1" data-bbox="1473 384 2130 427"> <tr> <td>3</td> <td>3</td> </tr> </table> <p>Six shared equally between two children is?</p> <p>Six divided equally between two children is?</p>	3	3
3	3			

Division as grouping

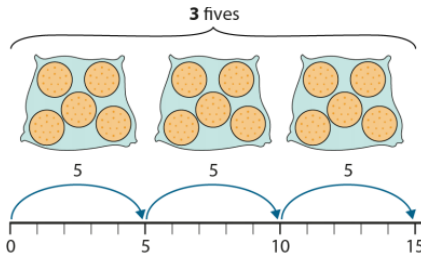
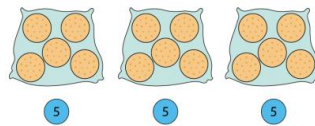
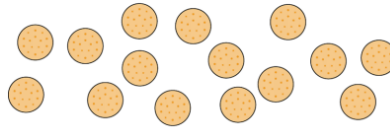
This can have an additive structure (additive grouping).

$$6 \div 2$$



Drawing pictures to represent the maths story.

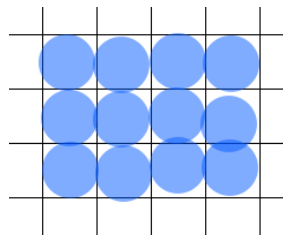
There are fifteen biscuits. If I put them into bags of five, how many bags will I need?



$$5 + 5 + 5 = 15$$
$$15 \div 5 = 3$$

Drawing numberline alongside pictures.

Linking to inverse through arrays



$$12 \div 3 = 4 \quad 12 \div 3 = 4$$
$$4 \times _ = 12 \quad _ \times 3 = 12$$

Recording the abstract to match the pictures in the story.

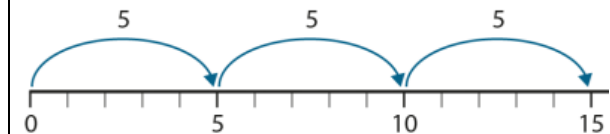
There are fifteen biscuits. If I put them into bags of five, how many bags will I need?

$$15 \div 5$$

- 'One bag of five is five.'
 - 'Two bags of five are ten.'
 - 'Three bags of five are fifteen.'

 - 'Fifteen is divided into groups of five. There are three groups.'
- $$15 \div 5 = 3$$
- 'Fifteen divided into groups of five is equal to three.'
 - 'So, we need three bags.'

Abstract number line on its own.

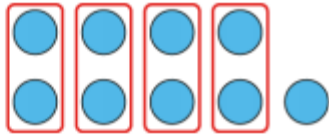


$$5 + 5 + 5 = 15$$

$$15 \div 5 = 3$$

2 digit divided by 1 digit with remainders

Using concrete equipment to show remainders.

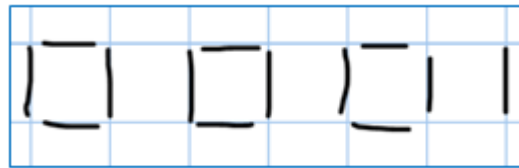
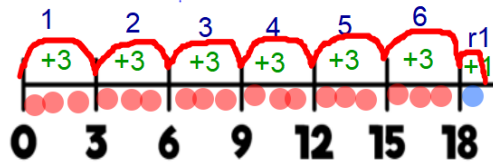


Use of lollipop sticks to form wholes

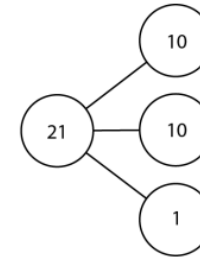


Using additive grouping on a numberline, with pictures.

$$19 \div 3 = 6 \text{ remainder } 1$$

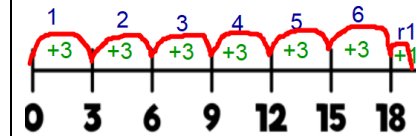


Use of the part whole and bar model to show the abstract concept and solve problems.



17			
5	5	5	2

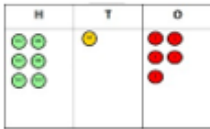
Additive grouping on a numberline using abstract numbers. $19 \div 3 =$



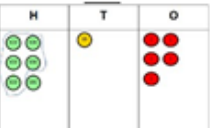
Using the bus stop method for trickier division that is not easily solved through known facts.

Using grouping and counters. Key language for grouping - How many groups of X can we make with X hundreds. This can also be done using sharing.

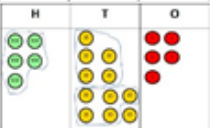
$$615 \div 5$$



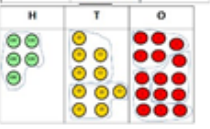
Step 1: make 615



Step 2: Circle your groups of 5



Step 3: Exchange 1H for 10T and circle groups of 5



Step 4: exchange 1T for 10ones and circles groups of 5

The concrete model can be represented pictorially until the children no longer need to do this.

$$\begin{array}{r} 123 \\ 5 \overline{) 615} \end{array}$$

Through the use of the bus stop method remainders need to be interpreted as fractions

$$\begin{array}{r} 07\frac{2}{8} \\ 8 \overline{) 58} \end{array} \quad \begin{array}{l} 2 \div 8 \\ 7 \text{ and } \frac{2}{8} \end{array}$$

And decimals.

$$\begin{array}{r} 07.125 \\ 8 \overline{) 57.1000} \end{array}$$

Fluency variation, different ways to ask children to solve division calculations. Children are encouraged to choose the most efficient way to solve each problem.

Using the part whole model below, how can you divide 615 by 5 without using the 'bus stop' method.



Balancing problems

$48 \div \underline{\quad} = 12 = \underline{\quad} \div 5$

$715 \div \underline{\quad} = 143$

I have £615 and share it equally between five bank accounts. How much will be in each account.

615 pupils need to be put into 5 groups. How many will be in each group?

Function machine

6	$\div 10 =$	
74		
8.3		

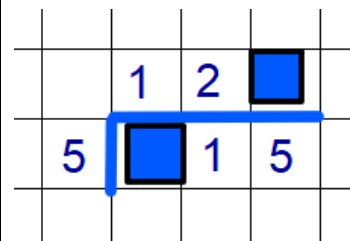
$$5 \overline{)615}$$

$615 \div 5 =$

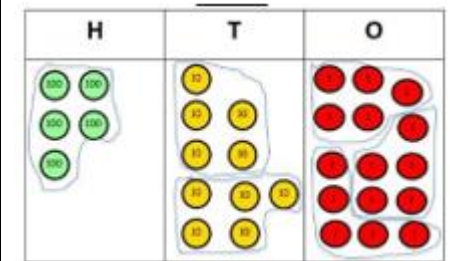
$\underline{\quad} = 615 \div 5$

How many fives go into 615?

Missing number problems



What's the calculation?
What's the answer?



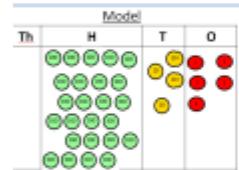
Long Division

Concrete



$$\begin{array}{r} 0212 \\ 12 \overline{)2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

$2544 \div 12$
How many groups of 12 thousands do we have? None



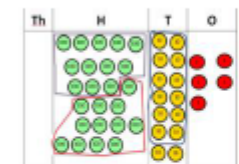
Exchange 2 thousand for 20 hundreds.



$$\begin{array}{r} 02 \\ 12 \overline{)2544} \\ \underline{24} \\ 1 \end{array}$$

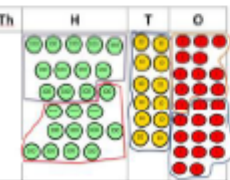
How many groups of 12 are in 25 hundreds? 2 groups. Circle them.

We have grouped 24 hundreds so can take them off and we are left with one.



$$\begin{array}{r} 021 \\ 12 \overline{)2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 2 \end{array}$$

Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2.



Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2

Pictorial

Children to represent the counters pictorially and record the subtractions beneath.

Abstract

$$12 \overline{)2544}^0$$

Step one- exchange 2 thousand for 20 hundreds so we now have 25 hundreds.

$$12 \overline{)2544}^{02}$$

Step two- How many groups of 12 can I make with 25 hundreds? The 24 shows the hundreds we have grouped. The one is how many hundreds we have left.

$$12 \overline{)2544}^{021}$$

Exchange the one hundred for 10 tens. How many groups of 12 can I make with 14 tens? The 14 shows how many tens I have, the 12 is how many I grouped and the 2 is how many tens I have left.

$$12 \overline{)2544}^{0212}$$

Exchange the 2 tens for 20 ones. The 24 is how many ones I have grouped and the 0 is what I have left.